NUMERICAL ANALYSIS

Assignment -8 (week 8)

Total Marks - 25

Posted on - 11/9/2017 (Monday);

To be submitted on or before-20/9/2017 (Wednesday), 23.59 hours.

Problems on

- Bisection Method
- Newton-Raphson Method

INSTRUCTIONS

- This is a question paper cum answer booklet.
- Take a print out of this.
- Present the details of the computations of the solution of each problem **which you will have to show** in the space provided at the bottom of the page.
- Fill in the answers in the space provided below each question.
- Scan the booklet and submit it as a pdf file before the deadline for evaluation.

1. Consider Newton's method and apply it to $f(x) = 0$, where $f(x)$ is a polynomial of degree 2, $f(x) = ax^2 + bx + c$. Find the coefficients of the polynomial so that
(a) $f(0) = 2$
(b) for $x_0 = 0$, $x_1 = 2$ and x_2 is undefined.
Fill in the blanks:
(i) $c =$ _____; (ii) $f'(0) =$ _____; (iii) $b =$ _____;
(iv) $a =$ _____ . (4 marks)

Show your work for the solution of problem 1 in the space provided below.

2. Use Newton-Raphson's method to approximate the solution to $x + e^x = 0$ with an error of atmost 10^{-4}.
Fill in the blanks:
Starting with an initial approximation x_0 as $x_0 = 1$,
(a) the value of n for which $|p_n - p_{n-1}| < 10^{-4}$ is satisfied is _____,
(b) an approximation to the root of $f(x) = 0$ with error less than 10^{-4} is,

$p_n =$ _____ . (6 marks)

Show your work for the solution of problem 2 in the space provided below.

3. Apply the bisection method to $f(x) = x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ with the initial interval $[a_1, b_1] = [2, 3]$. Perform 3 steps and find p_3.
Fill in the blanks:
(a) $p_1 =$ _____; (b) $f(a_1) =$ _____; (c) $f(b_1) =$ _____;
(d) $f(p_1) =$ _____; (e) $p_2 =$ _____; (f) $f(p_2) =$ _____;
(g) $p_3 =$ _____. (3 marks)

Show your work for the solution of problem 3 in the space provided below.

4. (a) Use the bisection method to find solutions correct to within 10^{-2} for $f(x) = x^3 - 7x^2 + 14x - 6 = 0$ on $[0, 1]$.
(b) Find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-3} to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 4]$.
Fill in the blanks:
(a) An approximation to a root of $f(x) = 0$ lying in the interval $[0, 1]$ correct to the desired degree of accuracy is _____,

(c) the number of iterations to ensure accuracy is _____ . (4+3=7 marks)

Show your work for the solution of problem 4 in the space provided below.

5. Use Newton's method to find the only real root of the equation $x^3 - x - 1 = 0$ correct to 9 decimal places.
Fill in the blanks:
(a)If the function $f(x)$ changes sign between $x = a$ and $x = b$, then
(i) $a =$ _____ (ii) $b =$ _____.

(b) the only real root of the equation $f(x) = 0$ correct to 9 decimal places is = _____ . (1+4=5 marks)

Show your work for the solution of problem 5 in the space provided below.